Logarithm Questions

Q1)

(a) Solve $5^x = 6$. [2]

(b) Solve $e^x(2e^x - 1) = 10$. [4]

(c) Solve the simultaneous equations

$$\frac{27^x}{\sqrt{9^y}} = 3,$$

$$\log_2 x - 2 = \log_2 y.$$ [5]

2)

(a) Sketch the graph of $y = \frac{2}{e^{2x}}$. [2]

(b) Solve the equation $\log_3(3x^2 - 6) - \log_{\sqrt{3}}(x-1) = 1$. [4]

(c) Given that $y = 2^x$, express $2^{3x-1} - 4^{3x} + 8^{x+1}$ in terms of y. [3]

3)

Solve the following equations

(i) $1 + \log_2 7 = 2\log_2(x-5)$ [3]

(ii) $\lg(7x-1) + \lg(x+2) = 2$ [3]

4)

(a) Express $\frac{\sqrt{3}+1}{3-\sqrt{3}} + \frac{1}{2+\sqrt{3}}$ in the form $a + b\sqrt{3}$, where a and b are rational numbers. [5]

(b) If $p = \log_{10} 2$ and $q = \log_{10} 7$, express $\log_{10}\sqrt[3]{\frac{25}{49}}$ in terms of p and q. [3]

5)

(a) Solve the equation $\log_4 y = 4 + 5\log_y 4$. [4]

(b) A student encounters the following question in an examination:

"Solve the equation $9^y + 5(3^y - 10) = 0$".

The following line shows the first step of the student's solution.

$$\lg 9^y + \lg 5(3^y) - \lg 50 = 0$$

State whether the student's first step is correct or wrong.

If it is correct, complete the steps for the student.

Otherwise, show the correct solution. [5]

6)

(a) ***Without using a calculator***, evaluate $\log_3 2 \times \log_8 9$. [3]

(b) By using an appropriate substitution, solve the equation

$$3e^{2x} - e^x = 4.$$ [3]

7)

(i) Given that $\log_p a = r$ and $\log_p b = s$, express $\log_{ab} p$ in terms of r and s. [2]

(ii) Solve the equation $3e^x + 11e^{\frac{1}{2}x} = 20$. [4]

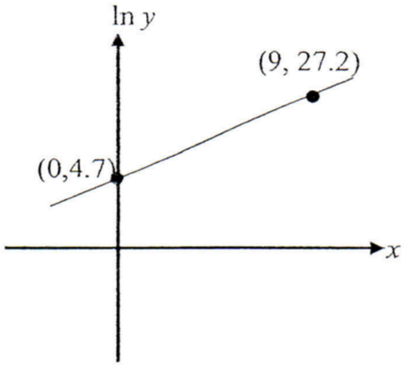
(iii) The population of a certain bacteria, P, present at time t hours after being observed initially is given by the formula $P = 480 + 300e^{0.5t}$.

(a) Calculate the initial population of the bacteria. [1]

(b) Find the population of the bacteria 12 hours after being observed initially and leave your answer correct to the nearest thousand. [1]

(c) Calculate the time taken for the bacteria to reach a population of 2700. Leave your answer correct to the nearest hour. [2]

8)



The variables x and y are connected by the equation $y = ab^x$, where a and b are constants. Experimental values of x and y were obtained. The diagram above shows the straight line graph, passing through the points (0, 4.7) and (9, 27.2), obtained by plotting $\ln y$ against x.

Estimate

(i) the value, to 2 significant figures, of a and of b, [6]

(ii) the value of y when $x = 2$. [2]

9)

(a) Solve $2\log_7 p = 3 + \log_p 49$. [5]

(b) Given that $\log_3 x = a$ and $\log_9 y = b$, express in terms of a and b,

(i) $\log_x 9y$, [4]

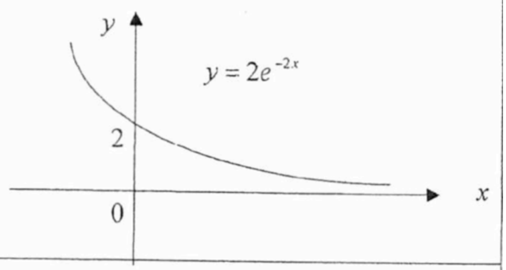
(ii) $x^3 y$. [3]

Answer

1)

2a	$x = 1.11(3\text{s.f.})$
2b	$x = 0.916(3\text{s.f.})$
2c	$y = \frac{1}{11} \qquad x = 4(\frac{1}{11}) = \frac{4}{11}$

2)

6a	
6b	$x = 1.5$
6c	$8\frac{1}{2}y^3 - y^6$

3)

3.(i)

$$1 + \log_2 7 = 2\log_2(x-5)$$
$$\log_2 2 + \log_2 7 = 2\log_2(x-5)$$
$$2 \times 7 = (x-5)^2$$
$$x^2 - 10x + 11 = 0 \quad \text{(M1)}$$
$$x = \frac{-(-10) \pm \sqrt{(-10)^2 - 4(1)(11)}}{2(1)}$$
$$= 1.258 \quad or \quad 8.742$$
$$= 1.26 \quad or \quad 8.74$$

(A1) (A1)

(rejected)

3.(ii)

$$\lg(7x-1) + \lg(x+2) = 2$$
$$\lg(7x-1)(x+2) = \lg 100$$
$$(7x-1)(x+2) = 100$$
$$7x^2 + 13x - 2 = 100 \quad \text{(M1)}$$
$$7x^2 + 13x - 102 = 0$$
$$(x-3)(7x+34) = 0$$
$$x = 3 \text{ or } x = -\frac{34}{7} \text{ (rejected)} \quad \text{(A1)}$$
$$\therefore x = 3$$

(A1)

4)

7(a)

$\frac{\sqrt{3}+1}{3-\sqrt{3}}+\frac{1}{2+\sqrt{3}}=\frac{(\sqrt{3}+1)(2+\sqrt{3})+(3-\sqrt{3})}{(3-\sqrt{3})(2+\sqrt{3})}$	[M1]	
$=\frac{(2\sqrt{3}+3+2+\sqrt{3})+3-\sqrt{3}}{6+3\sqrt{3}-2\sqrt{3}-3}$	[M1]	
$=\frac{2\sqrt{3}+8}{3+\sqrt{3}}\times\frac{3-\sqrt{3}}{3-\sqrt{3}}$	[M1]	
$=\frac{6\sqrt{3}-6+24-8\sqrt{3}}{9-3}$	[M1]	
$=\frac{18-2\sqrt{3}}{6}$		
$=3-\frac{1}{3}\sqrt{3}$	[A1]	

7(b)

$\log_{10}\sqrt[3]{\frac{25}{49}}=\log_{10}\left(\frac{25}{49}\right)^{\frac{1}{3}}$		
$=\frac{1}{3}[\log_{10}25-\log_{10}49]$	[M1]	Use of laws of log
$=\frac{1}{3}[2\log_{10}5-2\log_{10}7]$		
$=\frac{2}{3}\left[\log_{10}\frac{10}{2}-\log_{10}7\right]$	[M1]	5=10/2
$=\frac{2}{3}[\log_{10}10-\log_{10}2-q]$		
$=\frac{2}{3}[1-p-q]$	[A1]	

5)

8(a)

$\log_4 y=4+5\log_y 4$		
$\log_4 y=4+5\left(\frac{\log_4 4}{\log_4 y}\right)$	[M1]	Change of base
Let $\log_4 y=x$,		
$x=4+\frac{5}{x}$		
$x^2-4x-5=0$		
$(x-5)(x+1)=0$		
$x=5$ or $x=-1$		
$\log_4 y=5$; $\log_4 y=-1$	[M1]	Change to index form
$y=4^5$; $y=4^{-1}$	[M1]	For both answers
$=1024$; $=\frac{1}{4}$	[A1]	

8(b)

The step is wrong.	[B1]	
$9^y+5(3^y-10)=0$		
$3^{2y}+5(3^y-10)=0$		
Let $u=3^y$,		
$u^2+5(u-10)=0$	[M1]	
$u^2+5u-50=0$		
$(u+10)(u-5)=0$		
$u=-10$ or $u=5$		
$3^y=-10$ (rej) ; $3^y=5$	[M1]	For both and reject −10
$y=\frac{\lg 5}{\lg 3}$	[M1]	Apply log
$=1.46$ (3s.f)	[A1]	

6)

(a) $\frac{2}{3}$ (b) $\ln\frac{4}{3}$

7)

			marks
11	(i)	$\log_{ab} p = \frac{\log_p p}{\log_p ab}$ $= \frac{1}{\log_p a + \log_p b}$ $= \frac{1}{r+s}$	M1 for chang of base A1
	(ii)	$3e^{2\left(\frac{x}{2}\right)} + 11e^{\left(\frac{x}{2}\right)} = 20$ Let $y = e^{\frac{x}{2}}$ $3y^2 + 11y - 20 = 0$ $(y+5)(3y-4) = 0$ $y = -5$ or $y = \frac{4}{3}$ $e^{\frac{x}{2}} = -5$ (reject) Or $e^{\frac{x}{2}} = \frac{4}{3}$ $\frac{x}{2} = \ln\frac{4}{3}$ $x = 0.575(to 3sf)$	M1 for substitution and forming the quadratic eqn M1 factorising and solving A1 A1
	(iii)	$p = 480 + 300e^{0.5t}$	
	(a)	t = 0, $p = 480 + 300 = 780$	B1
	(b)	$p = 480 + 300e^{0.5(12)}$ = 121508.638= 122000(to the nearest thousand)	B1
	(c)	$2700 = 480 + 300e^{0.5t}$ $e^{0.5t} = \frac{2220}{300}$ $0.5t = \ln 7.4$ $t = \frac{\ln 7.4}{0.5} = 4.00296 = 4 hours$	M1 A1

8)

(i)		$a \approx 110$ and $b \approx 12$
(ii)		$y \approx 1630 (3s.f.)$

9)

(i)		$p \approx 0.378$ or $p = 49$
(ii)	**(a)**	$\log_x 9y = \frac{2+2b}{a}$
	(b)	3^{3a+2b}