Class Registration Number

Name: ____________________________

Class	Registration Number

CHIJ KATONG CONVENT

END-OF-YEAR EXAMINATION 2010

ADDITIONAL MATHEMATICS

SECONDARY THREE EXPRESS **4038**

Classes: Sec 303, 304, 305, 306 **Duration: 2 hours 30 minutes**

Additional materials: Writing paper
Graph paper

READ THESE INSTRUCTIONS FIRST

Write your name, class and index number in the spaces provided at the top of this page and on all the work you hand in. Write in dark blue or black pen. You may use a soft pencil for any diagrams or graphs. Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** questions.

Write your answers on the separate writing paper provided.

Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question. The use of a scientific calculator is expected, where appropriate. You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.

The number of marks is given in brackets [ ] at the end of each question or part question. The total number of marks for this paper is 100.

FOR EXAMINER'S USE	
TOTAL MARKS	______ 100

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial expansion

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \dots\dots + \binom{n}{r}a^{n-r}b^r + \dots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\dots\dots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2\sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

$$\sin A + \sin B = 2\sin\frac{1}{2}(A+B)\cos\frac{1}{2}(A-B)$$

$$\sin A - \sin B = 2\cos\frac{1}{2}(A+B)\sin\frac{1}{2}(A-B)$$

$$\cos A + \cos B = 2\cos\frac{1}{2}(A+B)\cos\frac{1}{2}(A-B)$$

$$\cos A - \cos B = -2\sin\frac{1}{2}(A+B)\sin\frac{1}{2}(A-B)$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

$$\Delta = \frac{1}{2}ab\sin C$$

1 Find the range of values of c for which $x^2 + 6x - 3 > 10x + c$, for all values of x. [3]

2 Solve the equation $e^{2x+1} + 8e = 6e^{x+1}$, giving your answer correct to 2 decimal places. [5]

3 When the expression $2x^3 + (a-3)x^2 + bx + 5$ is divided by $(x^2 - x - 2)$, the remainder is $3x + 5$. Find the values of a and b. [4]

4 Without using tables or calculators, find the value of k such that

$$\left(\frac{1}{\sqrt{6}} - \frac{\sqrt{24}}{3} + \frac{49}{\sqrt{294}}\right) \text{ x } \frac{3}{\sqrt{2}} = k\sqrt{3}$$ [4]

5 Express the simultaneous equations $kx + 3y = 5$, $4x + (5k-4)y = 10$ in matrix form.
Find and explain the values of k for which the above equations have

(a) an infinite number of solutions , [4]

(b) no solution. [3]

6 The roots of the equation $2x^2 + 7x + 3 = 0$ are α and β while the roots of the equation $hx^2 = 3x - k$ are $\alpha + 2$ and $\beta + 2$. Calculate the numerical values of h and k. [7]

7 Find the equation of the circle which passes through the points (2, 1) and (0, -3) and has centre lying on the line $3x + 5y - 3 = 0$. [8]

8 (a) In the expansion of $(2+x^2)^n$, the ratio of the coefficients of x^4 and x^6 is 2 : 1. Find the value of n. [4]

(b) In the expansion of $\left(x^2-\frac{2}{x}\right)^9$, find the term independent of x. [4]

9 Solve the equation $ln(4^y-4)-y\,ln2=ln3$. [6]

10 (a) Sketch the graph of $y=2-|2x-3|$ for $-1\le x\le 5$ and state its range. Find the values of x for which $y>1$. [4]

(b) The value of an investment at the beginning of 1990 was \$2000. Its value increases and after a period of t years is given by $V=2000(1.06)^t$. Find the year in which the investment will reach \$10 000. [4]

11 Find all the angles between 0° and 360° for which

(a) $3\sin x+2\operatorname{cosec}x=7$, [4]

(b) $3\tan(2x-30°)=2$. [4]

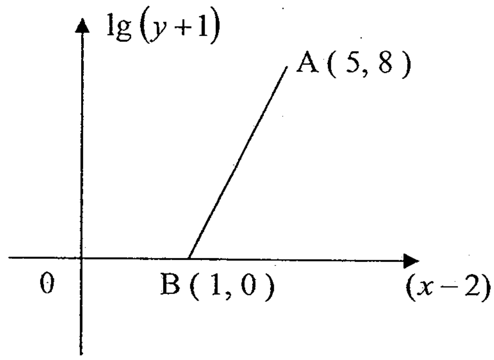
12 (a) Prove the identity $\frac{\sin^3\theta}{\cos^2\theta}+\sin\theta\equiv\tan\theta\sec\theta$. [3]

(b) (i) Sketch the graph of $y=3|sin\,2x|-1$ for $0\le x\le 2\pi$ and state the corresponding range of y. [3]

(ii) By drawing a suitable straight line on the graph, find the number of solutions of

$$3|sin\,2x|=1+\frac{x}{2\pi}$$ [2]

13 (a)



The above diagram shows part of the straight line graph obtained by plotting values of the variables indicated, together with the coordinates of the two points on the line. Express y in terms of x. [4]

(b) The table below shows experimental values of two variables x and y.

x	1.0	1.5	2.0	2.5	3.0	3.5
y	3.2	5.2	7.1	9.2	11.2	13.3

It is known that x and y are related by the equation $y = hx + k\sqrt{x}$, where h and k are constants.

(i) Using graph paper, draw the graph of $\frac{y}{\sqrt{x}}$ against $\sqrt{x}$ and use your graph to estimate the value of h and of k. [6]

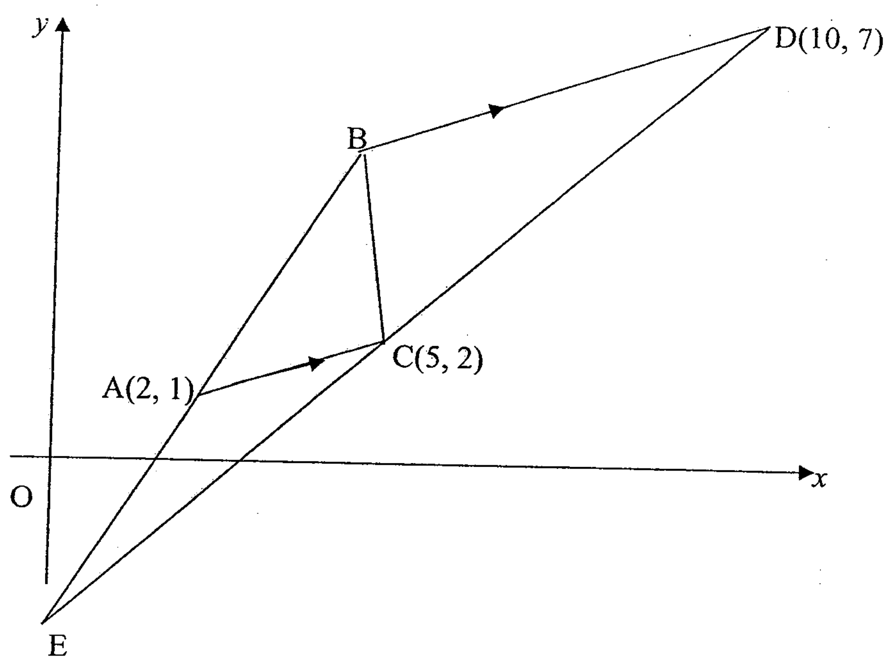
(ii) Use your graph to estimate the value of x when $y = 5\sqrt{x}$. [1]

(iii) Hence, by drawing a suitable straight line, estimate the value of x and of y which satisfy the simultaneous equations

$$y = hx + k\sqrt{x},$$
$$y = -4\sqrt{x} + 9x.$$
[3]

[Turn over

14 **Solutions to this question by accurate drawing will not be accepted.**



The diagram shows a triangle ABC in which A is the point (2, 1), C is the point (5, 2) and angle $ACB = 90^\circ$. The line BD is parallel to AC and D is the point (10, 7). The lines BA and DC are extended to meet at E.

Find

(i) the equation of BD and of BC, [4]

(ii) the coordinates of B, [2]

(iii) the coordinates of the vertex F of the parallelogram ABDF, [2]

(iv) the ratio of the area of the triangle EAC to the area of the trapezium ABDC. [2]

END OF PAPER

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1.

$x^2-4x-3-c>0$

$B^2-4AC<0$

$16-4(1)(-3-c)<0$

$28+4c<0$

$c<-7$

2

$e^{2x}\times e+8e-6e^x\times e=0$

$e^{2x}+8-6e^x=0$

$y^2-6y+8=0$

$(y-4)(y-2)=0$

$y=4, y=2$

$e^x=4$

$x=1.39(2dp)$

$e^x=2$

$x=0.69(2dp)$

3.

$f(2) = 11$

$f(-1) = 2$

$2a + b = 1 - - - - - - - (1)$

$a - b = 2 - - - - - - - (2)$

$a = 1, b = -1$

4.

$$\left(\frac{1}{\sqrt{6}} - \frac{2\sqrt{6}}{3} + \frac{7}{\sqrt{6}}\right)\left(\frac{3}{\sqrt{2}}\right) = \left(\frac{8}{\sqrt{6}} - \frac{2\sqrt{6}}{3}\right)\left(\frac{3}{\sqrt{2}}\right) = \frac{24}{\sqrt{12}} - \frac{2\sqrt{6}}{\sqrt{2}}$$

$$= \frac{24}{\sqrt{4 \times 3}} - 2\sqrt{3}$$

$$= \frac{12\sqrt{3}}{3} - 2\sqrt{3}$$

$$= 2\sqrt{3}$$

$k = 2$

5a.

$$\begin{vmatrix} k & 3 \\ 4 & 5k-4 \end{vmatrix} = 0$$

$$k(5k-4)-12=0$$

$$(5k+6)(k-2)=0$$

$$k=-\frac{6}{5} \qquad k=2$$

k = 2, the two lines are the same

$\therefore$ k = 2, is for infinitely many solutions

5b. for k = $-\frac{6}{5}$, the two lines are parallel $\therefore$ no solution.

6.

$$\alpha+\beta=-\frac{7}{2}------(1)$$

$$\alpha\beta=\frac{3}{2}------(2)$$

$$\alpha+\beta=\frac{3}{h}-4------(3)$$

$$\alpha\beta+2(\alpha+\beta)+4=\frac{k}{h}-----(4)$$

from (3)

$$-\frac{7}{2}=\frac{3}{h}-4$$

$h=6$

$k=-9$

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7. midpt (1,-1)

gradient of $\perp$ *line* = $-\frac{1}{2}$

equation of perpendicular line

$$y=-\frac{1}{2}x-\frac{1}{2}------(1)$$

given equation

$$y=\frac{3}{5}-\frac{3}{5}x------(2)$$

solving (1) and (2)

x = 11, y = -6.

Centre (11,-6)

Radius = $\sqrt{130}$

Equation $x^2+y^2-22x+12y+27=0$

8a.

$$^{n}C_{r}2^{n-r}x^{2r}$$

$$r=2, r=3$$

$$^{n}C_{2}2^{n-r} :^{n} C_{3}2^{n-3}$$

$$=2:1$$

$$\frac{\frac{n(n-1)(n-2)}{6}}{\frac{n(n-1)}{2}}\times 2^{n-3-n+2}=\frac{1}{2}$$

$$n=5$$

8b.

$$^{9}C_{r}(x^{2})^{9-r}(-2x^{-1})^{r}$$

$$=^{9} C_{r}(x^{18-2r})(-2)^{r}\left(x^{-r}\right)$$

$$18-3r=0$$

$$r=6$$

$$ans: 5376$$

9.

$$\ln\left(4^{y}-4\right)-\ln 2^{y}=\ln 3$$

$$\ln\frac{4^{y}-4}{2^{y}}=\ln 3$$

$$\frac{2^{2y}-4}{2^{y}}=3$$

$$A^{2}-3A-4=0$$

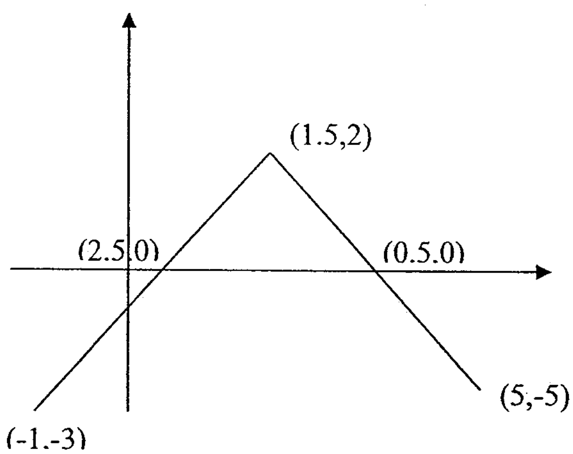
$$(A-4)(A+1)=0$$

$$A=4$$

$$y=2$$

$$A=-1(rejected)$$

10a.



y=1,   x=2, x=1,   1<x<2

10b. $10000 = 2000\,(1.06)^t$

t = 27.62

ans: 1990 + 27 = 2017.

11a.

$$3\sin x+\frac{2}{\sin x}=7$$

$$3A+\frac{2}{A}=7$$

$$3A^2-7A+2=0$$

$$(3A-1)(A-2)=0$$

$$A=\frac{1}{3},A=2$$

$$\sin x=\frac{1}{3},\sin x=2$$

$$x=19.5^o,160.5^o$$

11b. $\tan\ (2x-30^0)=\frac{2}{3}$

$$\alpha=33.69^0$$

$$2x-30^0=33.69,\ 213.69,\ 393.69,\ 573.69$$

$$x=31.8,\ 121.8,211.8,\ 301.8$$

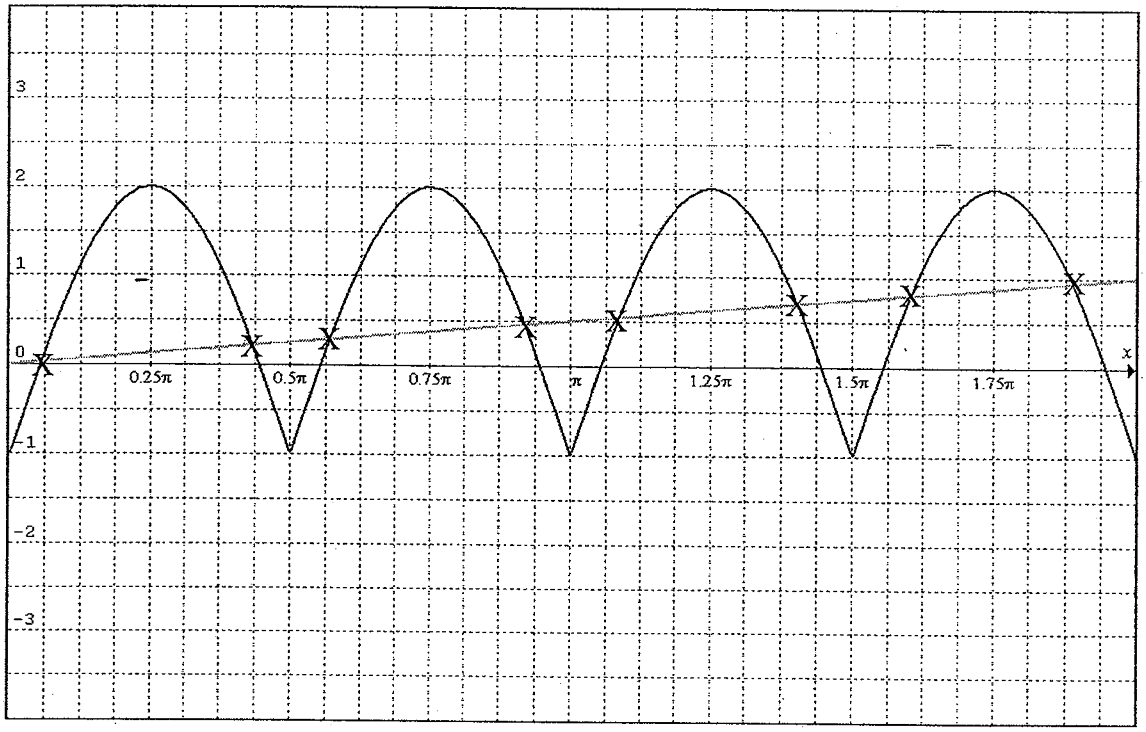
12a.

$$\frac{\sin^3\theta+\cos^2\theta\sin\theta}{\cos^2\theta}=\frac{\sin^3\theta+\left(1-\sin^2\theta\right)\sin\theta}{\cos^2\theta}=\frac{\sin\theta}{\cos^2\theta}=\frac{\sin\theta}{\cos\theta\cos\theta}$$

$$=\tan\theta\ \ \sec\theta$$

proven

12bi.



$-1 \le y \le 2$

12bii 8 solutions

x	0	π	2π
y	0	0.5	1

13a . m = 2
c = -2

$$Y = 2X - 2$$
$$\lg(y+1) = 2(x-2) - 2$$
$$\lg(y+1) = 2x - 6$$
$$y + 1 = 10^{2x-6}$$
$$y = 10^{2x-6} - 1$$

13b. i. gradient = h = 4.5
k = Y-int = -1.3

$\sqrt{x}$	1.0	1.22	1.41	1.58	1.73	1.87
$\frac{y}{\sqrt{x}}$	3.2	4.25	5.02	5.82	6.47	7.11

ii. $\frac{y}{\sqrt{x}} = 5, \quad \sqrt{x} = 1.4$

$x = 1.96$

iii. $Y = 9X - 4$

X	0.6	1
Y	1.4	5

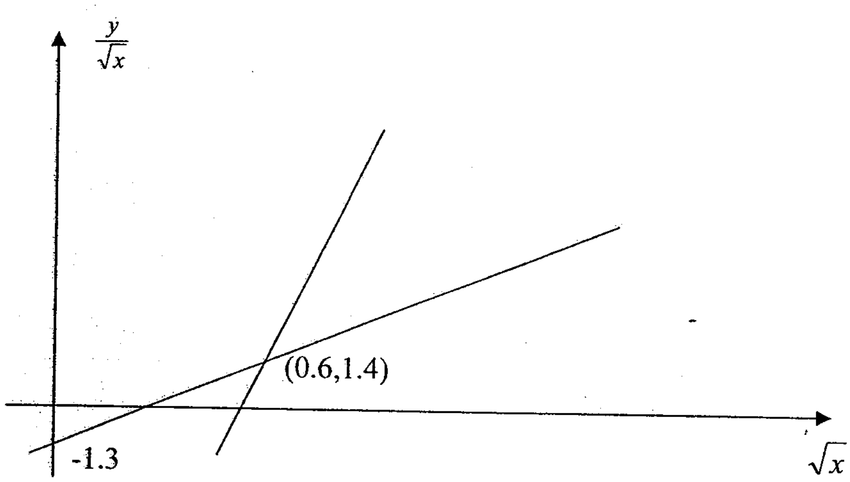
Point of intersection (0.6,1.4)

$\sqrt{x} = 0.6$

$x = 0.36$

$\frac{y}{\sqrt{x}} = 1.4$

$y = 0.84$



14. gradient of BD = $\frac{1}{3}$

y -7 = $\frac{1}{3}(x-10)$

y = $\frac{1}{3}x+\frac{11}{3}$ equation of BD

gradient of BC = -3

equation of BC:

y = -3x+17

ii. solving the two equations : B (4,5)

$$(6,4)=\left(\frac{x+4}{2},\frac{y+5}{2}\right)$$

iii. $x=8, y=5$

$F(8,5)$

iv. BD = $\sqrt{40}=2\sqrt{10}$
AC = $\sqrt{10}$

Ratio is 1 : 2

$$\frac{area\text{ of }\Delta AEC}{area\text{ of }\Delta EBD}=(\frac{1}{2})^2$$

$$\frac{area\text{ of }\Delta AEC}{area\text{ of trapezium}ABDC}=(\frac{1}{4-1})=\frac{1}{3}$$